Quantum Interference and Duality

Kiyohide NOMURA

Department of Physics

December 21, 2016

Quantum Physics(Mechanics)

- Basic notion of Quantum Physics: "Wave-Particle Duality"
  - Light (electromagnetic wave)
    - Light as wave
      Interference, Diffraction, Polarization
    - Light as a particle: Photon
      Photoelectric effect, Compton effect

Quantum Physics(Mechanics)

- Basic notion of Quantum Physics: "Wave-Particle Duality"
  - Light (electromagnetic wave)
    - Light as wave
      Interference, Diffraction, Polarization
    - Light as a particle: Photon
      Photoelectric effect, Compton effect
  - Electron
    - Electron as a particle
      Mass-to-charge ratio
      Elementary electric charge (Millikan's oil drop experiment)
    - Electron as wave
      Davisson-Germer experiment (1923-1927)

Light as wave (Interference)

Interference: a phenomenon in which two waves superpose to form a resultant wave of greater, lower, or the same amplitude
MOVIE: Interference of waves from two point sources.

Light as wave (Interference)

Interference: a phenomenon in which two waves superpose to form a resultant wave of greater, lower, or the same amplitude
MOVIE: Interference of waves from two point sources.

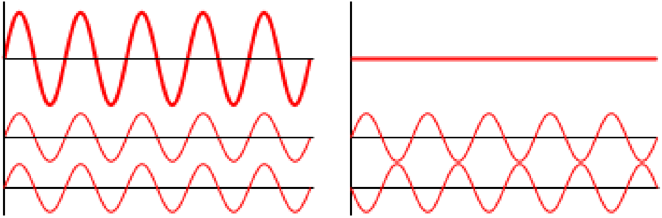


Figure: Left: constructive interference; Right: destructive interference

Light as wave (Interference of thin film)

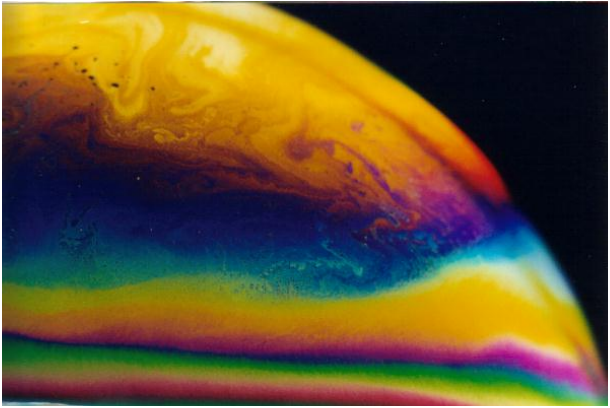


Figure: Interference pattern on a soap bubble

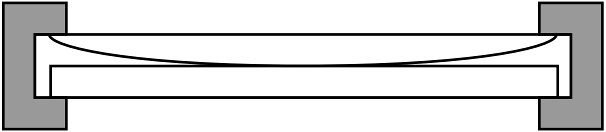
Light as wave (Newton's ring)

Another (and quantitative) example of interference:
Newton's ring (1717)

Light as wave (Newton's ring)

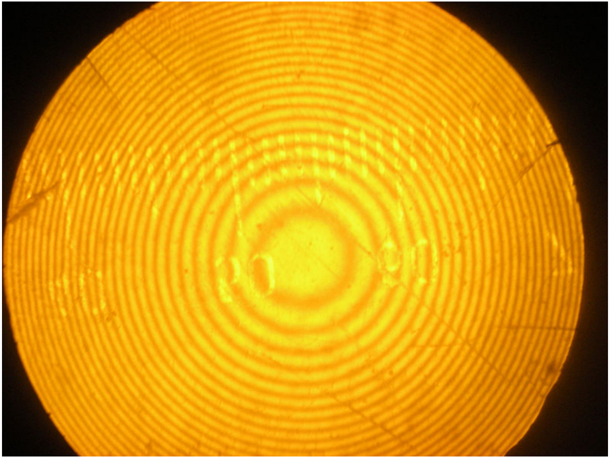
Another (and quantitative) example of interference:
Newton's ring (1717)

an interference pattern created by placing a very slightly convex curved glass (lens) on an optical flat glass.

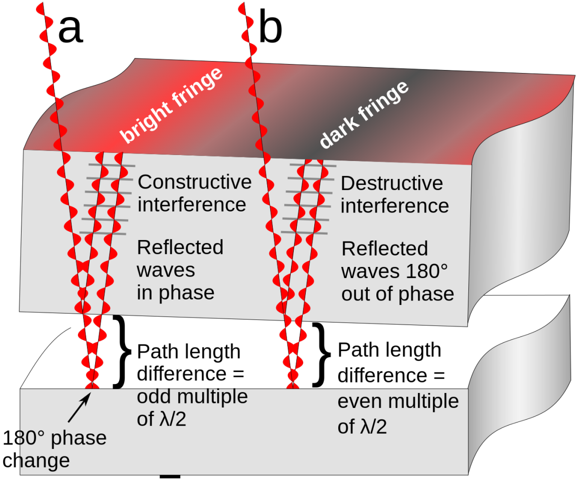


Light as wave (Newton's ring)

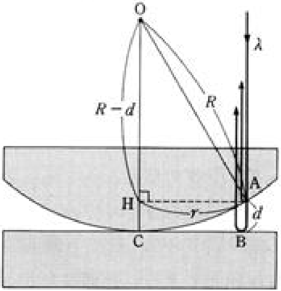
When viewed with monochromatic light, Newton's rings appear as a series of concentric, alternating bright and dark rings centered at the point of contact between the two surfaces.



Light as wave (Newton's ring)



Light as wave (Newton's ring)



R: the radius of curvature of the glass lens
d: the vertical distance between the glass lens and the flat glass

$$R^2 = (R-d)^2 + r^2 = R^2 - 2Rd + d^2 + r^2 \qquad (1)$$

$$\therefore r^2 = 2Rd\left(1 - \frac{d}{2R}\right) \approx 2Rd \qquad (\because d \ll R) \qquad (2)$$

Light as wave (Newton's ring)

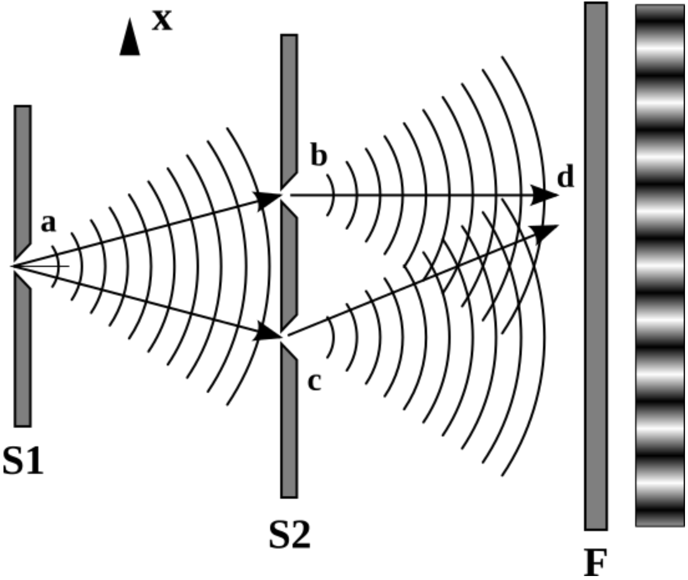
- Reflection at the glass-air boundary causes no phase shift
- Reflection at the air-glass boundary causes a half-cycle phase (π) shift

Thus, when the distance $2d$ is $m\lambda$(λ: the wavelength) , the two waves interfere destructively.

The radius r of the Nth dark ring is given by

$$r = \sqrt{m\lambda R}, \qquad (m = 0, 1, 2, \cdots) \tag{3}$$

Light as wave: Double Slit



Light as wave: Double Slit

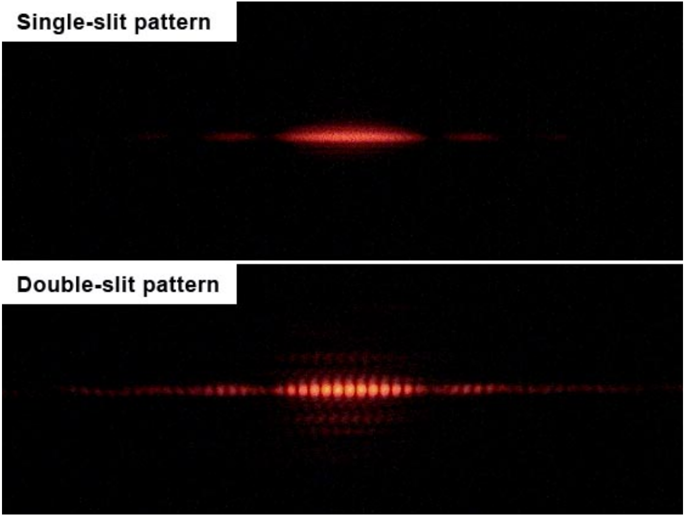


Figure: Up Single slit Down: Double Slit distance between slits 0.7mm

Light as wave: Diffraction

Diffraction: various phenomena which occur when a wave encounters an obstacle or a slit

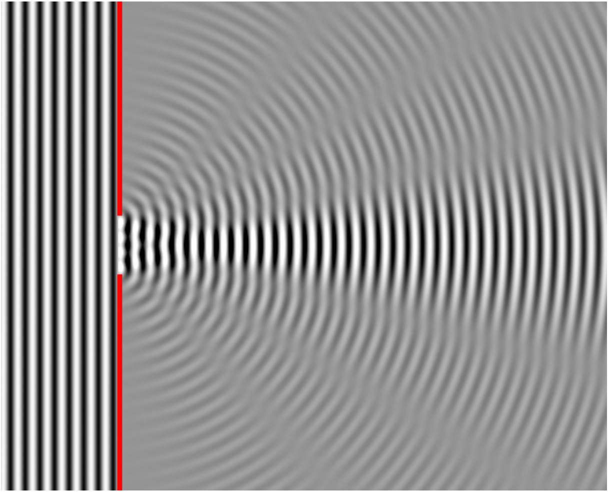


Figure: diffraction pattern from a slit of width four wavelengths with an incident plane wave

Light as wave: Diffraction

MOVIE: diffraction pattern from a slit of width equal to five times the wa

Light as wave: Diffraction

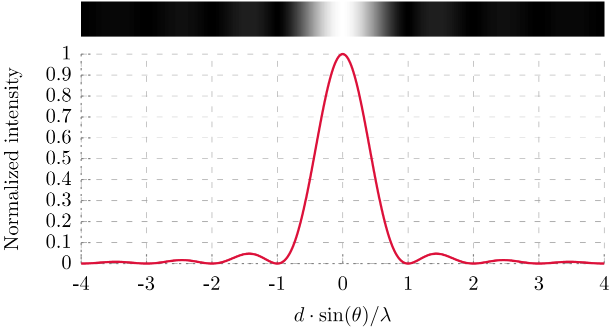
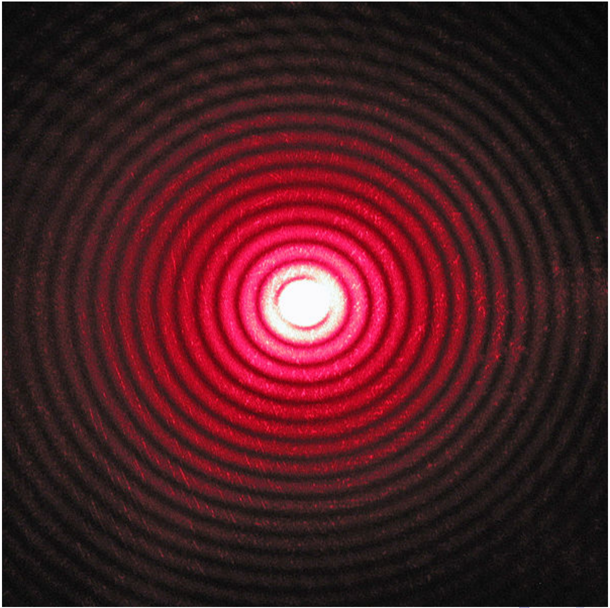
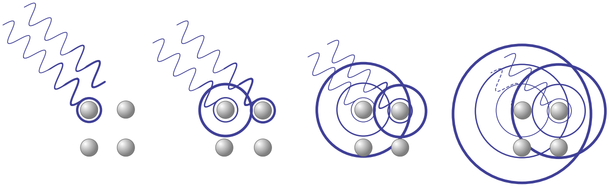


Figure:

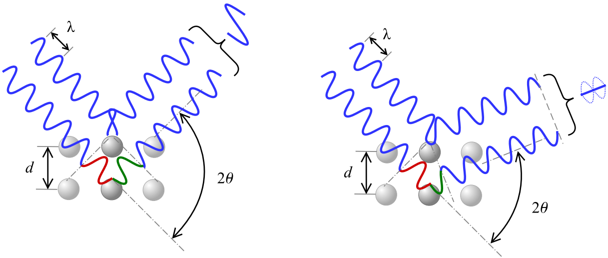
Light as wave: Diffraction



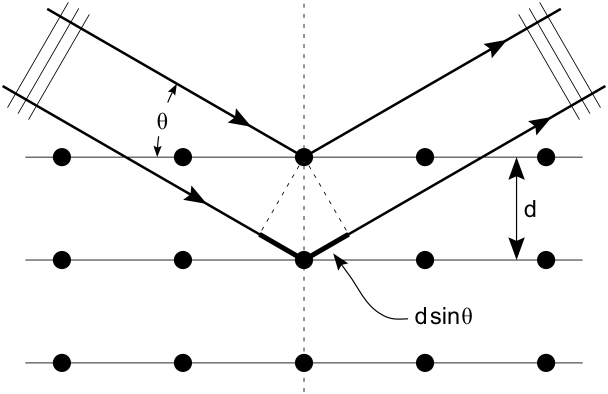
Light as wave: Bragg's law (X ray diffraction)



Light as wave: Bragg's law (X ray diffraction)



Light as wave: Bragg's law (X ray diffraction)



$$2d\sin\theta = n\lambda \quad (n = 1, 2, \cdots) \tag{4}$$

- λ: the wavelength of incident wave.
- d: separation between planes of lattice points
- θ: the scattering angle

Light as wave: Bragg's law (X ray diffraction)

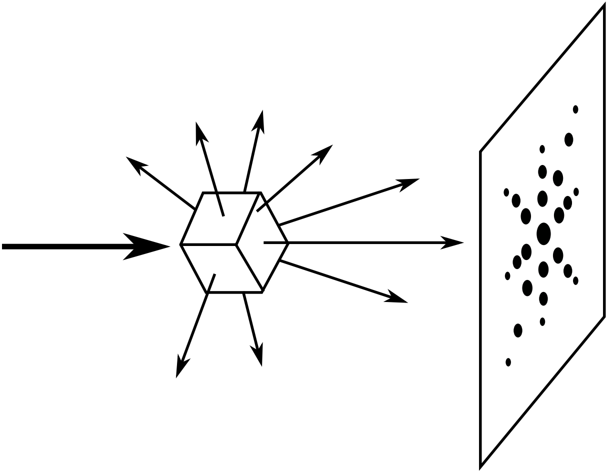
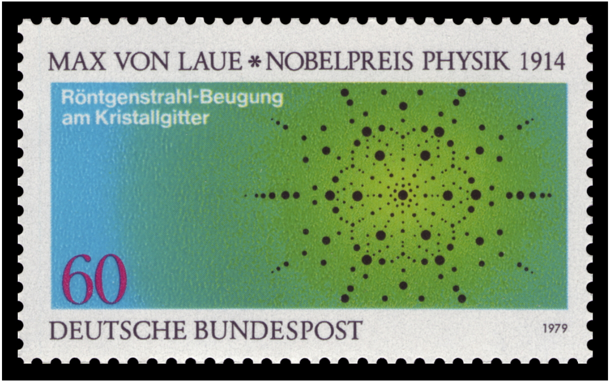


Figure: Laue pattern

Light as wave: Bragg's law (X ray diffraction)



Light as a particle: Photoelectric effect

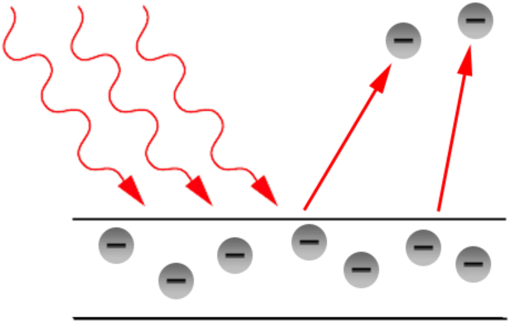
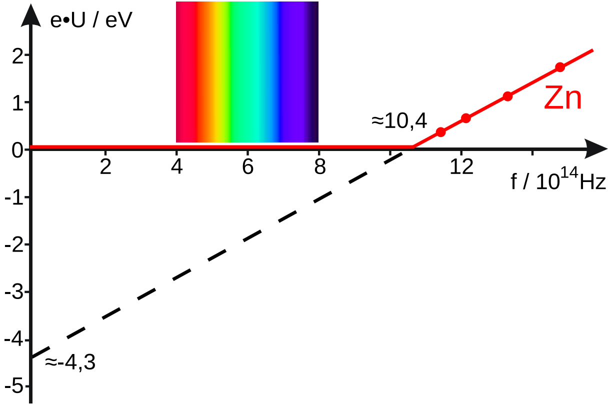


Figure: the production of electrons or other free carriers when light is shone onto a material

Light as a particle: Photoelectric effect

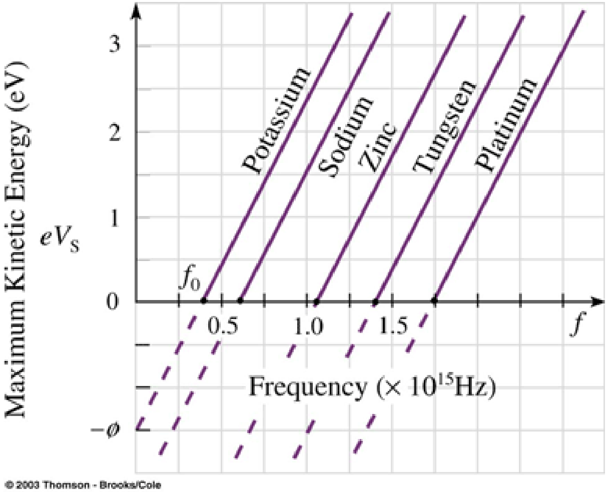


$$\boxed{E_{\text{kin}} = h\nu - W} \quad (5)$$

- h: Planck's constant ($6.62606957 \times 10^{-34} \text{m}^2\text{kg/s}$)
- ν: the frequency of the incident photon

Light as a particle: Photoelectric effect

The work function W (which gives the minimum energy required to remove a delocalised electron from the surface of the metal) is different between materials.



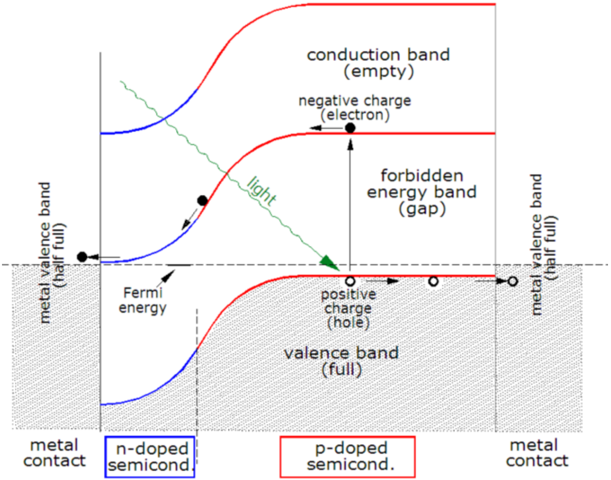
Light as a particle: Photoelectric effect

Internal photoemission

- Solar cell
- CCD
- photosynthesis in plants

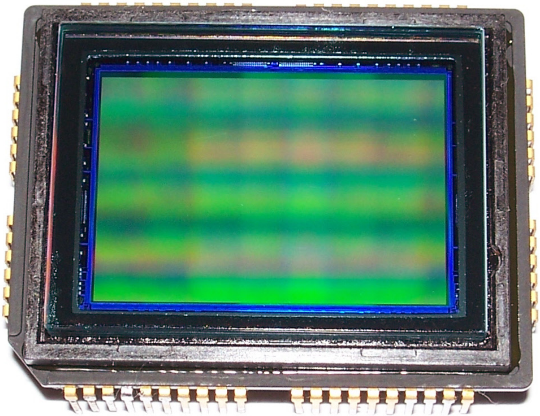
Light as a particle: Photoelectric effect

Theory of the solar cell



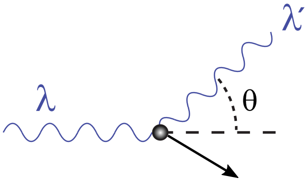
Light as a particle: Photoelectric effect

CCD



Light as a particle: Compton scattering

scattering between a photon (X-ray or gamma ray) and a charged particle (electron)



$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta) \qquad (6)$$

- λ: the initial wavelength, λ': the wavelength after scattering,$\lambda' > \lambda$
- θ: the scattering angle
- h: the Planck constant $(6.62606957(29) \times 10^{-34}\mathrm{m^2 kg/s})$
- m_e: the electron rest mass $(9.10938291(40) \times 10^{-31}\mathrm{kg})$
- c: the speed of light $(2.99792458 \times 10^{8}\mathrm{m/s})$

Light as a particle: Compton effect

Energy quanta relation(quantum)

$$E = h\nu \tag{7}$$

The relation of the wavelength λ ,frequency ν , and light speed c

$$c = \lambda\nu \tag{8}$$

The relation between the energy E and the momentum $\boldsymbol{p}$ of the electromagnetic wave

$$E = c|\boldsymbol{p}| \tag{9}$$

We obtain the relation between the momentum and the wavelength

$$\boxed{p = \frac{h}{\lambda}} \tag{10}$$

Electron as a particle (Mass-to-charge ratio)

$$\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \tag{11}$$

(Lorentz force law)

$$\boldsymbol{F} = m\boldsymbol{a} = m\frac{d\boldsymbol{v}}{dt} \tag{12}$$

(Newton's second law of motion)
Combining the two previous equations yields:

$$\boxed{\left(\frac{m}{q}\right)\boldsymbol{a} = \boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}} \tag{13}$$

Electron as a particle (Mass-to-charge ratio)

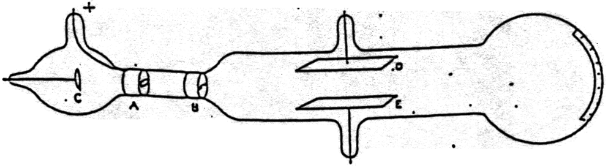


Figure: The cathode ray tube by Thomson

Cathode rays were emitted from the cathod C, passed through slits A (the anode) and B (grounded), then through the electric field generated between plates D and E, finally impacting the surface at the far end.

Electron as a particle (Mass-to-charge ratio)

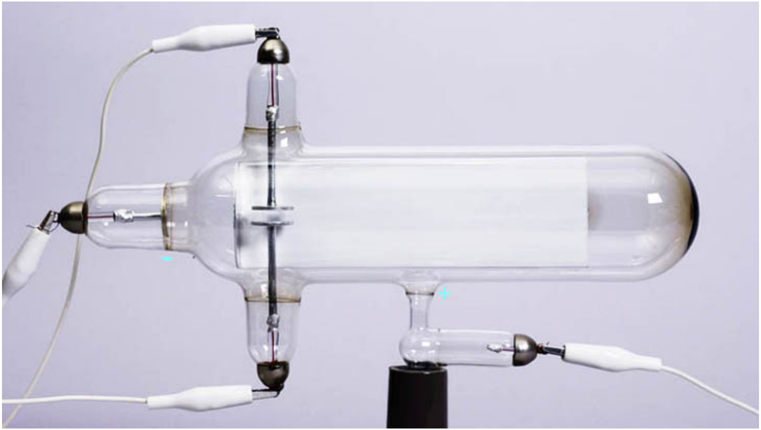


Figure: Crookes tube

Electron as a particle (Mass-to-charge ratio)

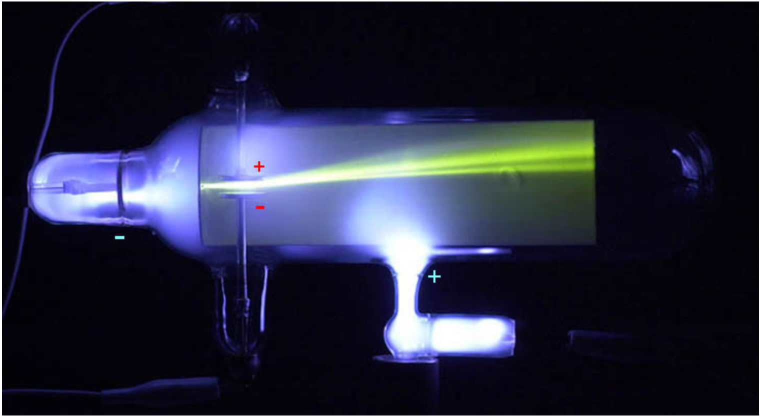


Figure: Crookes tube with the electric field

The cathode ray was deflected by the electric field

Electron as a particle (Mass-to-charge ratio)

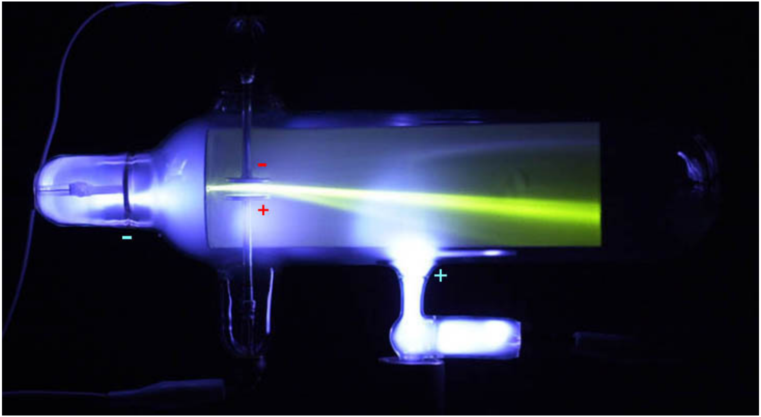


Figure: Crookes tube with the electric field

Electron as a particle (Mass-to-charge ratio)

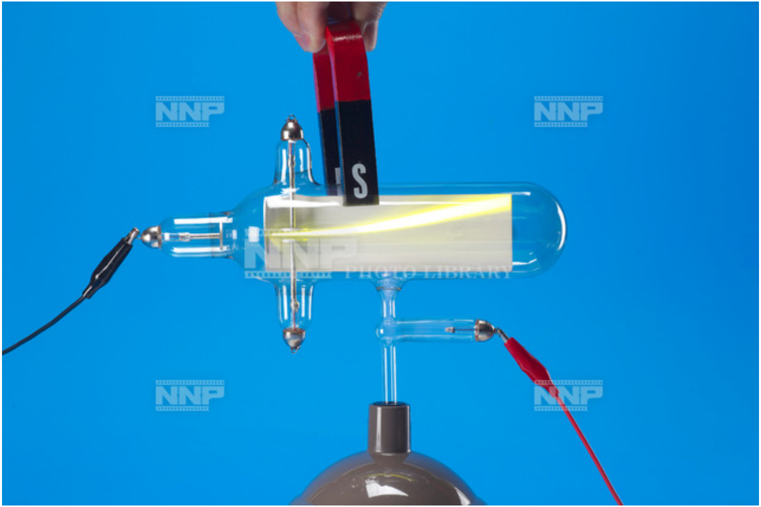


Figure: Crookes tube under the magnetic field

Electron as a particle (Charge)

Millikan's oil drop experiment(1909)

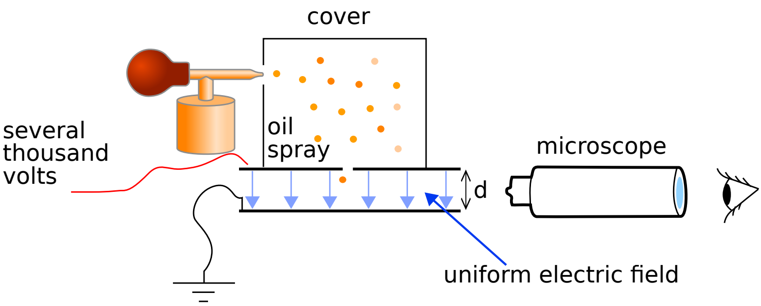


Figure: Millikan's oil drop experiment

Electron as a particle (Charge)

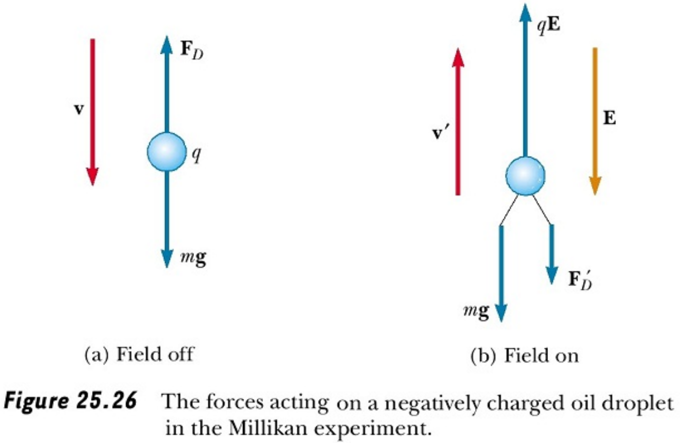


Figure: Millikan's oil drop experiment

Electron as a particle (Charge)

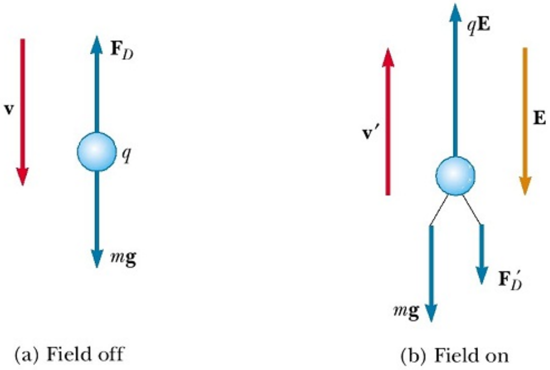


Figure 25.26 The forces acting on a negatively charged oil droplet in the Millikan experiment.

Figure: Millikan's oil drop experiment

electric charge $-1.602176565(35) \times 10^{-19}\mathrm{C}$

Electron as a wave

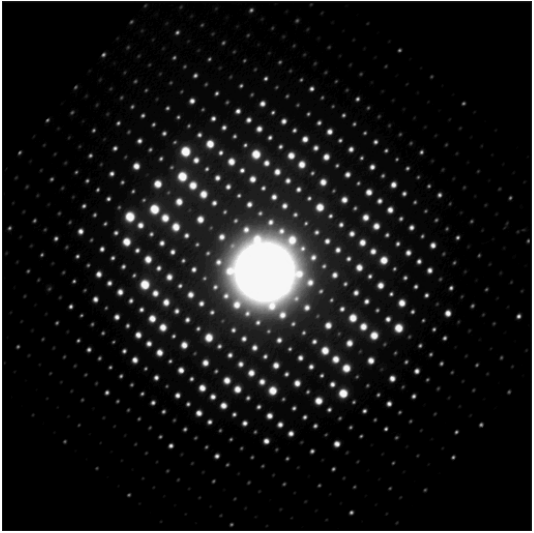


Figure: electron diffraction by Ta_2O_5

Electron as a wave

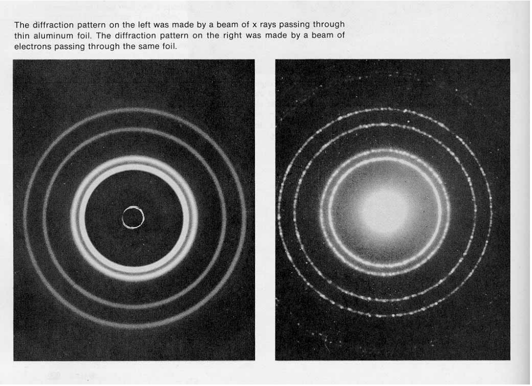


Figure: left: diffraction by X-ray right: diffraction by electron (Al film)

The similarity of the two diffraction patterns means that the electron has a wave character as X-ray.

Electron as a wave (de Broglie wave)

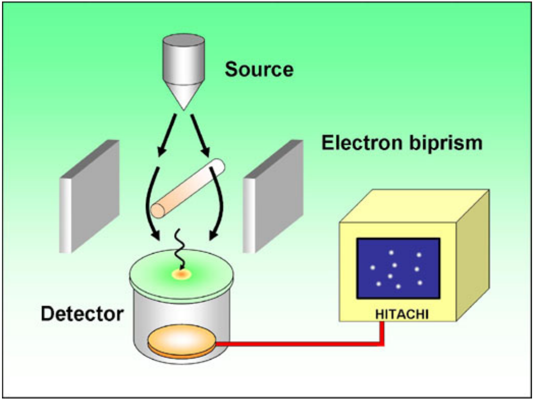
L. de Broglie proposed (1924) that the electron (in general matter) with momentum p behaves as a wave with wavelength

$$\boxed{\lambda = \frac{h}{p}} \tag{14}$$

from the analogy with the photon.

Electron as a wave

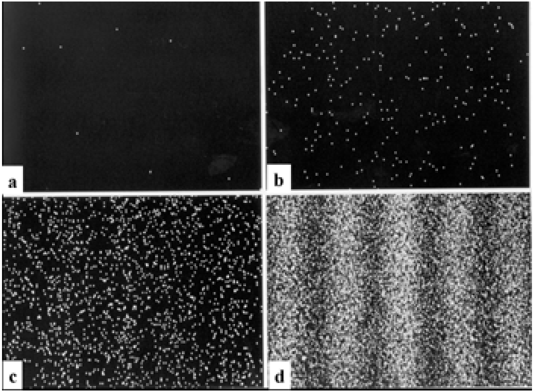
Double slit experimentA. Tonomura, 1982



Electrons pass through a device called the "electron biprism", which consists of two parallel plates and a fine filament at the center

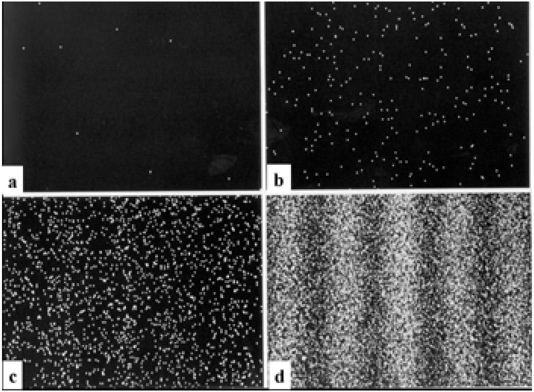
The filament is thinner than 1 micron.

Electron as a wave



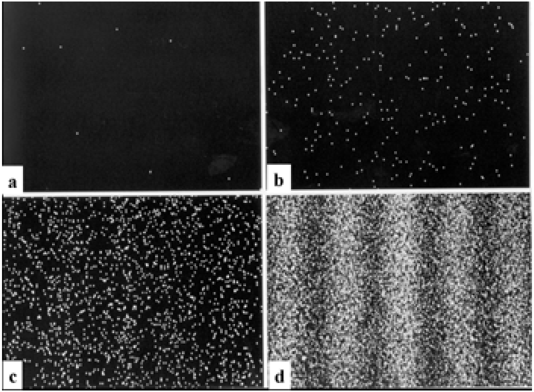
1. At the beginning of the experiment, we can see that bright spots (electrons) begin to appear here and there at random positions (Fig. 2 (a) and (b))

Electron as a wave



1. At the beginning of the experiment, we can see that bright spots (electrons) begin to appear here and there at random positions (Fig. 2 (a) and (b))
2. Electrons are detected one by one as particles

Electron as a wave



1. At the beginning of the experiment, we can see that bright spots (electrons) begin to appear here and there at random positions (Fig. 2 (a) and (b))
2. Electrons are detected one by one as particles
3. Clear interference fringes can be seen in the last scene of the experiment after 20 minutes (Fig. 2(d)).

Electron as a wave

MOVIE: the electron double slit experiment

Electron as a wave

Supplement for double slit experiment

1. Electron source: Field-emission gun
   more coherent and with up to three orders of magnitude greater current density or brightness than can be achieved with conventional thermionic emitters
2. These electrons were accelerated to 50,000 V (50keV), the speed is about 40 % of the speed of the light, i. e., it is 120,000 km/second.
3. There is no more than one electron in the microscope at one time, since only 10 electrons are emitted per second.
4. No interaction between electrons.

Electron as a wave

Supplement for double slit experiment

1. Electron source: Field-emission gun
   more coherent and with up to three orders of magnitude greater current density or brightness than can be achieved with conventional thermionic emitters
2. These electrons were accelerated to 50,000 V (50keV), the speed is about 40 % of the speed of the light, i. e., it is 120,000 km/second.
3. There is no more than one electron in the microscope at one time, since only 10 electrons are emitted per second.
4. No interaction between electrons.

Single electron is enough to create a quantum interference!

Neutron as a wave

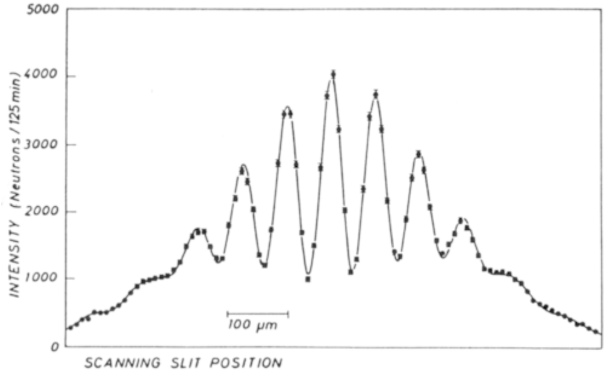


Figure: A double-slit interference pattern made with neutrons.

Neutron as a wave

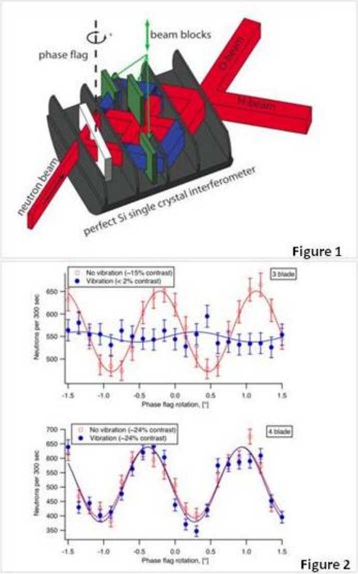
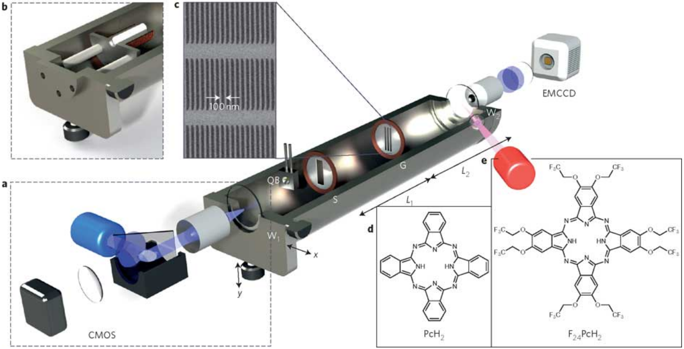


Figure: neutron interferrometer

Molecule as a wave: Phthalocyanine



Molecule as a wave: Phthalocyanine

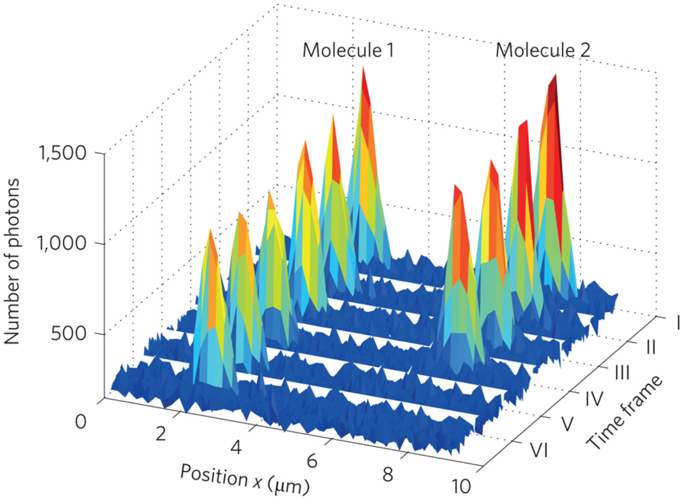


Figure:

Molecule as a wave: Phthalocyanine

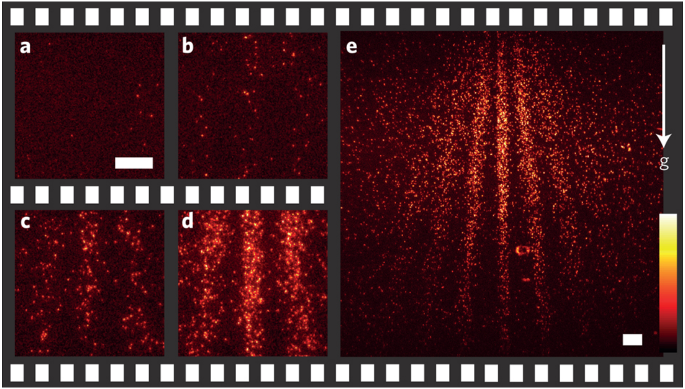


Figure:

Molecule as a wave: Phthalocyanine