第1章　Conformal theories in d dimensions

1.1　Conformal group in d dimensions

1. ミスプリ

   5ページ，下から5行目

   - 誤:$\omega^{\mu}{}_{\nu}\epsilon^{\nu}\partial_{\mu}$ → 正:$\omega^{\mu}{}_{\nu}x^{\nu}\partial_{\mu}$
   - 誤:$b^{\mu}(x^2\partial_{\mu} - 2x^{\mu}x \cdot \partial)$ → 正:$b^{\mu}(x^2\partial_{\mu} - 2x_{\mu}x \cdot \partial)$
     なお，$x \cdot \partial \equiv x^{\alpha}\partial_{\alpha}$

2. リー代数を満たすこと：

   (a) 和:

   $$\hat{A} + \hat{B} \tag{1.1}$$

   について演算が閉じていること，

   (b) スカラー倍: $a\hat{A}$ (a は実数) について演算が閉じていること，

   (c) 交換子:

   $$\begin{aligned} [\hat{A}, \hat{B}] &\equiv \hat{A}\hat{B} - \hat{B}\hat{A}, \\ [\hat{A}, \hat{B}] &= -[\hat{B}, \hat{A}] \end{aligned} \tag{1.2}$$

   について演算が閉じていること，

   (d) Jacobi の恒等式

   $$[\hat{A}, [\hat{B}, \hat{C}]] + [\hat{B}, [\hat{C}, \hat{A}]] + [\hat{C}, [\hat{A}, \hat{B}]] = 0 \tag{1.3}$$

   を満たすことを確認すれば良い.

交換子について Jacobi の恒等式は自明.

(a) $a^\mu \partial_\mu$

i. 和：

$$a^\mu \partial_\mu + a'^\alpha \partial_\alpha = (a^\mu + a'^\mu)\partial_\mu \tag{1.4}$$

和について閉じている.

ii. スカラー倍:自明

iii. 交換子：

$$[a^\mu \partial_\mu, a'^\alpha \partial_\alpha] = a^\mu a'^\alpha [\partial_\mu, \partial_\alpha] = 0 \tag{1.5}$$

交換子について閉じている.

以上から，$a^\mu \partial_\mu$ はリー代数を満たしている.

(b) $\omega^\mu{}_\nu x^\nu \partial_\mu$

i. 和：

$$\omega^\mu{}_\nu x^\nu \partial_\mu + \omega'^\alpha{}_\beta x^\beta \partial_\alpha = (\omega^\mu{}_\nu + \omega'^\mu{}_\nu) x^\nu \partial_\mu, \tag{1.6}$$

ここで，係数は

$$\omega_{\mu\nu} + \omega'_{\mu\nu} = -(\omega_{\nu\mu} + \omega'_{\nu\mu}), \qquad (\omega_{\mu\nu} \equiv g_{\mu\beta}\omega^\beta_\nu) \tag{1.7}$$

で反対称なので，和について閉じている.

ii. スカラー倍:

iii. 交換子：

$$\begin{aligned}
&[\omega^\mu{}_\nu x^\nu \partial_\mu, \omega'^\alpha{}_\beta x^\beta \partial_\alpha] \\
&= \omega^\mu{}_\nu \omega'^\alpha{}_\beta [x^\nu \partial_\mu, x^\beta \partial_\alpha] \\
&= \omega^\mu{}_\nu \omega'^\alpha{}_\beta (x^\nu [\partial_\mu, x^\beta \partial_\alpha] + [x^\nu, x^\beta \partial_\alpha]\partial_\mu) \\
&= \omega^\mu{}_\nu \omega'^\alpha{}_\beta (x^\nu (x^\beta [\partial_\mu, \partial_\alpha] + [\partial_\mu, x^\beta]\partial_\alpha) + (x^\beta [x^\nu, \partial_\alpha] + [x^\nu, x^\beta]\partial_\alpha)\partial_\mu) \\
&= \omega^\mu{}_\nu \omega'^\alpha{}_\beta (x^\nu ([\partial_\mu, x^\beta]\partial_\alpha) + (x^\beta [x^\nu, \partial_\alpha])\partial_\mu) \\
&= \omega^\mu{}_\nu \omega'^\alpha{}_\beta (x^\nu (\delta^\beta_\mu \partial_\alpha) + x^\beta (-\delta^\nu_\alpha)\partial_\mu) \\
&= \omega^\mu{}_\nu \omega'^\alpha{}_\mu x^\nu \partial_\alpha - \omega^\mu{}_\nu \omega'^\nu{}_\beta x^\beta \partial_\mu \\
&= (\omega^\mu{}_\gamma \omega'^\delta{}_\mu - \omega^\delta{}_\nu \omega'^\nu{}_\gamma) x^\gamma \partial_\delta
\end{aligned} \tag{1.8}$$

ところで，係数部分の添字を整理したもの $(\omega^\mu{}_\gamma\omega'_{\delta_\mu}-\omega_{\delta_\mu}\omega'^\mu{}_\gamma)$ の，添字 γ と δ を入れ替えると

$$\begin{aligned}
&\omega^\mu{}_\delta\omega'_{\gamma\mu}-\omega_{\gamma_\mu}\omega'^\mu{}_\delta \\
&=\omega^\mu{}_\delta(-\omega'_{\mu\gamma})-(-\omega_{\mu\gamma})\omega'^\mu{}_\delta \\
&=\omega_{\mu\delta}(-\omega'^\mu{}_\gamma)-(-\omega^\mu{}_\gamma)\omega'_{\mu\delta} \\
&=(-\omega_{\delta\mu})(-\omega'^\mu{}_\gamma)-(-\omega^\mu{}_\gamma)(-\omega'_{\delta\mu}) \\
&=-(\omega^\mu{}_\gamma\omega'_{\delta_\mu}-\omega_{\delta_\mu}\omega'^\mu{}_\gamma) \qquad (1.9)
\end{aligned}$$

で反対称なので，交換子について閉じている．

以上から，$\omega^\mu{}_\nu x^\nu\partial_\mu$ はリー代数を満たしている．

(c) $\lambda x\cdot\partial$

i. 和:

$$\lambda x\cdot\partial+\lambda' x\cdot\partial=(\lambda+\lambda')x\cdot\partial \qquad (1.10)$$

和について閉じている．

ii. スカラー倍:自明

iii. 交換子:

$$\begin{aligned}
&[\lambda x\cdot\partial,\lambda' x\cdot\partial] \\
&=\lambda\lambda'[x^\mu\partial_\mu,x^\alpha\partial_\alpha] \\
&=\lambda\lambda'(x^\mu[\partial_\mu,x^\alpha\partial_\alpha]+[x^\mu,x^\alpha\partial_\alpha]\partial_\mu) \\
&=\lambda\lambda'(x^\mu(x^\alpha[\partial_\mu,\partial_\alpha]+[\partial_\mu,x^\alpha]\partial_\alpha)+(x^\alpha[x^\mu,\partial_\alpha]+[x^\mu,x^\alpha]\partial_\alpha)\partial_\mu) \\
&=\lambda\lambda'(x^\mu(\delta^\alpha_\mu\partial_\alpha)+(x^\alpha(-\delta^\mu_\alpha)\partial_\mu)=0 \qquad (1.11)
\end{aligned}$$

以上から，$\lambda x\cdot\partial$ はリー代数を満たしている．

(d) $b^\mu(x^2\partial_\mu-2x_\mu x\cdot\partial)$

i. 和:

ii. スカラー倍:

iii. 交換子:

$$\begin{aligned}
&[b^\mu(x^2\partial_\mu-2x_\mu x\cdot\partial),b'^\alpha(x^2\partial_\alpha-2x_\alpha x\cdot\partial)] \\
&=b^\mu b'^\alpha[(x^2\partial_\mu-2x_\mu x^\nu\partial_\nu),(x^2\partial_\alpha-2x_\alpha x^\beta\partial_\beta)]=0 \qquad (1.12)
\end{aligned}$$

1.2 Conformal algebra in 2 dimensions

1.3 Constraints of conformal invariance in d dimensions

ミスプリ：(1.13) 式の 1 行目
誤: $\langle\phi_1(x_1)\cdots\phi_\nu(x_n)\rangle$
正: $\langle\phi_1(x_1)\cdots\phi_n(x_n)\rangle$

第2章　Conformal theories in 2 dimensions

2.1　Correlation functions of primary fields

1. 13 ページ，4 行目 ミスプリ
2. (2.3) 式の 2 行目 ミスプリ

   誤: $h_1\partial\bar{\epsilon}(\bar{z}_1), \quad h_2\partial\bar{\epsilon}(\bar{z}_2)$

   正: $\bar{h}_1\partial\bar{\epsilon}(\bar{z}_1), \quad \bar{h}_2\partial\bar{\epsilon}(\bar{z}_2)$
3. 式 (2.3) から大局的共形変換で式（2．4）を導く

$$\left(\left(\epsilon(z_1)\partial_{z_1} + h_1\partial\epsilon(z_1)\right) + \left(\epsilon(z_2)\partial_{z_2} + h_2\partial\epsilon(z_2)\right)\right) G^{(2)}(z_i) = 0 \tag{2.1}$$

   (a) $\epsilon(z_i) = 1$

$$\left(\partial_{z_1} + \partial_{z_2}\right) G^{(2)}(z_i) = 0 \tag{2.2}$$

   より，$G^{(2)}(z_i) = G^{(2)}(z_1 - z_2)$

   (b) $\epsilon(z_i) = z_i$

$$\left(z_1\partial_{z_1} + h_1 + z_2\partial_{z_2} + h_2\right) G^{(2)}(z_i) = 0 \tag{2.3}$$

   より，$G^{(2)}(z_i) = C_{12} z_{12}^{-h_1-h_2}$

   (c) $\epsilon(z_i) = (z_i)^2$

$$\begin{aligned}
&\left(z_1^2\partial_{z_1} + 2h_1 z_1 + z_2^2\partial_{z_2} + 2h_2 z_2\right) G^{(2)}(z_i) \\
&= C_{12}\left(z_1^2\partial_{z_1} + 2h_1 z_1 + z_2^2\partial_{z_2} + 2h_2 z_2\right) z_{12}^{-h_1-h_2} \\
&= C_{12}\left((z_1^2 - z_2^2)(-h_1 - h_2) z_{12}^{-h_1-h_2-1} + (2h_1 z_1 + 2h_2 z_2) z_{12}^{-h_1-h_2}\right) \\
&= C_{12}\left((z_1 + z_2)(-h_1 - h_2) z_{12}^{-h_1-h_2} + (2h_1 z_1 + 2h_2 z_2) z_{12}^{-h_1-h_2}\right) \\
&= C_{12}\left(h_1 z_1 - h_2 z_1 - h_1 z_2 + h_2 z_2\right) z_{12}^{-h_1-h_2} \\
&= C_{12}(h_1 - h_2) z_{12}^{-h_1-h_2+1} = 0
\end{aligned} \tag{2.4}$$

より，

i. $h_1 \neq h_2$
   $C_{12} = 0$

ii. $h_1 = h_2 = h$
   $G^{(2)}(z_i) = C_{12} z_{12}^{-2h}$

4. 大局的共形変換で式（2．5）を導く

$$\left(\left(\epsilon(z_1)\partial_{z_1} + h_1 \partial\epsilon(z_1)\right) + \left(\epsilon(z_2)\partial_{z_2} + h_2 \partial\epsilon(z_2)\right) + \left(\epsilon(z_3)\partial_{z_3} + h_3 \partial\epsilon(z_3)\right)\right) G^{(3)}(z_i) = 0 \tag{2.5}$$

(a) $\epsilon(z_i) = 1$

$$\left(\partial_{z_1} + \partial_{z_2} + \partial_{z_3}\right) G^{(3)}(z_i) = 0 \tag{2.6}$$

より，$G^{(3)}(z_i) = G^{(3)}(z_{12}, z_{23}, z_{31})$

(b) $\epsilon(z_i) = z_i$

$$\left(z_1\partial_{z_1} + h_1 + z_2\partial_{z_2} + h_2 + z_3\partial_{z_3} + h_3\right) G^{(3)}(z_i) = 0 \tag{2.7}$$

に，$G^{(3)}(z_i) = C_{abc} z_{12}^a z_{23}^b z_{31}^c$ を代入すると

$$\begin{aligned} & C_{abc}\left(z_1\partial_{z_1} + h_1 + z_2\partial_{z_2} + h_2 + z_3\partial_{z_3} + h_3\right) z_{12}^a z_{23}^b z_{31}^c \\ & = C_{abc}\left(z_1(a z_{12}^{-1} - c z_{31}^{-1}) + h_1 + z_2(-a z_{12}^{-1} + b z_{23}^{-1}) + h_2 + z_3(-b z_{23}^{-1} + c z_{31}^{-1}) + h_3\right) z_{12}^a z_{23}^b z \\ & = C_{abc}\left(a + b + c + h_1 + h_2 + h_3\right) z_{12}^a z_{23}^b z_{31}^c = 0 \end{aligned} \tag{2.8}$$

これから，$a + b + c + h_1 + h_2 + h_3 = 0$ でなければならない．

(c) $\epsilon(z_i) = (z_i)^2$

$$\begin{aligned} & C_{abc}\left(z_1^2\partial_{z_1} + 2h_1 z_1 + z_2^2\partial_{z_2} + 2h_2 z_2 + z_3^2\partial_{z_3} + 2h_3 z_3\right) z_{12}^a z_{23}^b z_{31}^c \\ & = C_{abc}\left(z_1^2(a z_{12}^{-1} - c z_{31}^{-1}) + 2h_1 z_1 + z_2^2(-a z_{12}^{-1} + b z_{23}^{-1}) + 2h_2 z_2 + z_3^2(-b z_{23}^{-1} + c z_{31}^{-1}) + 2h_3 \\ & = C_{abc}\left((z_1^2 - z_2^2) a z_{12}^{-1} + (z_2^2 - z_3^2) b z_{23}^{-1} + (z_3^2 - z_1^2) c z_{31}^{-1} + (2h_1 z_1 + 2h_2 z_2 + 2h_3 z_3\right) z_{12}^a z_{23}^b z \\ & = C_{abc}\left((z_1 + z_2)a + (z_2 + z_3)b + (z_3 + z_1)c + 2h_1 z_1 + 2h_2 z_2 + 2h_3 z_3\right) z_{12}^a z_{23}^b z_{31}^c \\ & = C_{abc}\left(z_1(a + c + 2h_1) + z_2(a + b + 2h_2) + z_3(c + b + 2h_3)\right) z_{12}^a z_{23}^b z_{31}^c = 0 \end{aligned} \tag{2.9}$$

上記の式が成立する為には

$$a+c+2h_1=0 \tag{2.10a}$$
$$a+b+2h_2=0 \tag{2.10b}$$
$$c+b+2h_3=0 \tag{2.10c}$$

でなければならない．これと $a+b+c=-h_1-h_2-h_3$ を組み合わせると

$$a=-h_1-h_2+h_3 \tag{2.11a}$$
$$b=-h_2-h_3+h_1 \tag{2.11b}$$
$$c=-h_3-h_1+h_2 \tag{2.11c}$$

5. 15 ページ 4 行目

“In (2.6) the cross-ratio x is defined as $x = z_{12}z_{34}/z_{13}z_{24}$. We note that this cross-ratio is annihilated by the differential operator $\sum_{i=1}^{4}\epsilon(z_i)\partial_{z_i}$...” の意味

まず,

$$\partial_{z_1}x \equiv \partial_{z_1}\left(\frac{z_{12}z_{34}}{z_{13}z_{24}}\right) = \frac{z_{34}}{z_{13}z_{24}} - \frac{z_{12}z_{34}}{(z_{13})^2 z_{24}} = \frac{x}{z_{12}} - \frac{x}{z_{13}} \tag{2.12}$$

同様にして,

$$\begin{aligned}&\sum_{i=1}^{4}\epsilon(z_i)\partial_{z_i}x\\&=\epsilon(z_1)\left(\frac{x}{z_{12}}-\frac{x}{z_{13}}\right)+\epsilon(z_2)\left(-\frac{x}{z_{12}}-\frac{x}{z_{24}}\right)+\epsilon(z_3)\left(\frac{x}{z_{34}}+\frac{x}{z_{13}}\right)+\epsilon(z_4)\left(-\frac{x}{z_{34}}+\frac{x}{z_{24}}\right)\end{aligned} \tag{2.13}$$

が成り立つ.

(a) $\epsilon(z_i)=1$

$$\begin{aligned}&\sum_{i=1}^{4}\partial_{z_i}x\\&=\left(\frac{x}{z_{12}}-\frac{x}{z_{13}}\right)+\left(-\frac{x}{z_{12}}-\frac{x}{z_{24}}\right)+\left(\frac{x}{z_{34}}+\frac{x}{z_{13}}\right)+\left(-\frac{x}{z_{34}}+\frac{x}{z_{24}}\right)=0\end{aligned} \tag{2.14}$$

(b) $\epsilon(z_i) = z_i$

$$
\begin{aligned}
&\sum_{i=1}^{4} z_i \partial_{z_i} x \\
&= z_1 \left(\frac{x}{z_{12}} - \frac{x}{z_{13}} \right) + z_2 \left(-\frac{x}{z_{12}} - \frac{x}{z_{24}} \right) + z_3 \left(\frac{x}{z_{34}} + \frac{x}{z_{13}} \right) + z_4 \left(-\frac{x}{z_{34}} + \frac{x}{z_{24}} \right) \\
&= (z_1 - z_2)\frac{x}{z_{12}} + (-z_1 + z_3)\frac{x}{z_{13}} + (-z_2 + z_4)\frac{x}{z_{24}} + (z_3 - z_4)\frac{x}{z_{34}} \\
&= x - x - x + x = 0 \qquad (2.15)
\end{aligned}
$$

(c) $\epsilon(z_i) = z_i^2$

$$
\begin{aligned}
&\sum_{i=1}^{4} z_i^2 \partial_{z_i} x \\
&= z_1^2 \left(\frac{x}{z_{12}} - \frac{x}{z_{13}} \right) + z_2^2 \left(-\frac{x}{z_{12}} - \frac{x}{z_{24}} \right) + z_3^2 \left(\frac{x}{z_{34}} + \frac{x}{z_{13}} \right) + z_4^2 \left(-\frac{x}{z_{34}} + \frac{x}{z_{24}} \right) \\
&= (z_1^2 - z_2^2)\frac{x}{z_{12}} + (-z_1^2 + z_3^2)\frac{x}{z_{13}} + (-z_2^2 + z_4^2)\frac{x}{z_{24}} + (z_3^2 - z_4^2)\frac{x}{z_{34}} \\
&= x((z_1 + z_2) - (z_1 + z_3) - (z_2 + z_4) + (z_3 + z_4)) = 0 \qquad (2.16)
\end{aligned}
$$

以上より cross-ratio x が, 大局的な共形変換 $\epsilon(z_i)$ について $\sum_{i=1}^{4} \epsilon(z_i)\partial_{z_i} x = 0$ が分かる.

2.2 Radial quantization and conserved charges

1. カレントと保存電荷 Q (１７ページ，4 行目)

(a)

$$
\begin{aligned}
\int_V \partial_\mu j^\mu d^d x &= \int_V \partial_0 j^0 d^d x + \int \nabla \cdot \boldsymbol{j} d^d x \\
&= \int_V \partial_0 j^0 d^d x + \oint_{\partial V} \boldsymbol{j} \cdot d\boldsymbol{s} \qquad (2.17)
\end{aligned}
$$

(第 2 項の式変形でガウスの定理を使っている).
第 2 項 (表面項) は $\boldsymbol{j}(x)$ が無限遠で 0 になる場合 (または周期境界条件) には 0 となるので，$\partial_\mu j^\mu = 0$ ならば

$$
\partial^0 Q(x_0) = 0, \qquad (Q(t) = \int d^d x j_0(x)) \qquad (2.18)
$$

(b)

$$\delta_\epsilon A = \epsilon[Q, A] \tag{2.19}$$

ポアソン括弧をもちいて，無限小変換を書き直したもの

注意：ネーターカレントと無限小変換の母関数についてはやや微妙なところがある (高橋 康)

2. ストレス−エネルギーテンソル ($T_{\mu\nu}$)

カレント $j_\mu = T_{\mu\nu}\epsilon^\nu$($\epsilon^\nu$ は (1.2) 式)

(a) 並進不変性 $\epsilon^\mu = a^\mu$

$$\partial_\mu j^\mu = \partial^\mu j_\mu = \partial^\mu(T_{\mu\nu}a^\nu) = (\partial^\mu T_{\mu\nu})a^\nu = 0 \tag{2.20}$$

これが任意の $^\mu$ について成立するためには

$$\partial^\mu T_{\mu\nu} = 0 \qquad \text{(divergence free)} \tag{2.21}$$

(b) 回転不変性 $\epsilon^\mu = \omega^\mu{}_\nu x^\nu$

$$\begin{aligned}\partial_\mu j^\mu &= \partial^\mu j_\mu \\ &= \partial^\mu(T_{\mu\nu}\omega^\nu{}_\alpha x^\alpha) = \partial^\mu(T_{\mu\nu}\omega^{\nu\alpha}x_\alpha) \\ &= (\partial^\mu T_{\mu\nu})\omega^{\nu\alpha}x_\alpha + T_{\mu\nu}\omega^{\nu\alpha}(\partial^\mu x_\alpha) \\ &= T_{\mu\nu}\omega^{\nu\alpha}\delta^\mu_\alpha = T_{\alpha\nu}\omega^{\nu\alpha} = 0\end{aligned} \tag{2.22}$$

（上記で， $\partial^\mu T_{\mu\nu} = 0$ を使った.）$\omega^{\nu\alpha} = -\omega^{\alpha\nu}$ を考慮すると

$$T_{\alpha\nu} = T_{\nu\alpha} \qquad \text{(対称テンソル)} \tag{2.23}$$

(c) スケール不変性 $\epsilon^\mu = \lambda x^\mu$

$$\begin{aligned}\partial_\mu j^\mu &= \partial^\mu j_\mu \\ &= \partial^\mu(T_{\mu\nu}\lambda x^\nu) = \lambda((\partial^\mu T_{\mu\nu})x^\nu + T_{\mu\nu}(\partial^\mu x^\nu)) \\ &= \lambda T_{\mu\nu}\eta^{\mu\nu} = \lambda T_\mu{}^\mu = 0\end{aligned} \tag{2.24}$$

これが任意の λ について成り立つ為には

$$T_\mu{}^\mu = 0 \qquad \text{(traceless)} \tag{2.25}$$

3.

$$T_{\mu\nu} \to T'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu}T_{\alpha\beta}(x) \tag{2.26}$$

(a)

$$\begin{aligned}
T'_{zz} &= \frac{\partial x^\alpha}{\partial z}\frac{\partial x^\beta}{\partial z}T_{\alpha\beta} \\
&= \frac{\partial x^0}{\partial z}\frac{\partial x^0}{\partial z}T_{00} + \frac{\partial x^0}{\partial z}\frac{\partial x^1}{\partial z}T_{01} + \frac{\partial x^1}{\partial z}\frac{\partial x^0}{\partial z}T_{10} + \frac{\partial x^1}{\partial z}\frac{\partial x^1}{\partial z}T_{11} \\
&= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)T_{00} + \left(\frac{1}{2}\right)\left(\frac{1}{2i}\right)T_{01} + \left(\frac{1}{2i}\right)\left(\frac{1}{2}\right)T_{10} + \left(\frac{1}{2i}\right)\left(\frac{1}{2i}\right)T_{11} \\
&= \frac{1}{4}\left(T_{00} - iT_{01} - iT_{10} - T_{11}\right)
\end{aligned} \tag{2.27}$$

(ここで， $x^0 = \frac{1}{2}(z+\bar{z}), x^1 = \frac{1}{2i}(z-\bar{z})$ を使った). さらに $T_{01} = T_{10}$ をつかうと

$$T'_{zz} = \frac{1}{4}\left(T_{00} - 2iT_{10} - T_{11}\right) \tag{2.28}$$

(b)

$$\begin{aligned}
T'_{z\bar{z}} &= \frac{\partial x^\alpha}{\partial z}\frac{\partial x^\beta}{\partial \bar{z}}T_{\alpha\beta} \\
&= \frac{\partial x^0}{\partial z}\frac{\partial x^0}{\partial \bar{z}}T_{00} + \frac{\partial x^0}{\partial z}\frac{\partial x^1}{\partial \bar{z}}T_{01} + \frac{\partial x^1}{\partial z}\frac{\partial x^0}{\partial \bar{z}}T_{10} + \frac{\partial x^1}{\partial z}\frac{\partial x^1}{\partial \bar{z}}T_{11} \\
&= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)T_{00} + \left(\frac{1}{2}\right)\left(-\frac{1}{2i}\right)T_{01} + \left(\frac{1}{2i}\right)\left(\frac{1}{2}\right)T_{10} + \left(\frac{1}{2i}\right)\left(-\frac{1}{2i}\right)T_{11} \\
&= \frac{1}{4}\left(T_{00} + iT_{01} - iT_{10} + T_{11}\right)
\end{aligned} \tag{2.29}$$

$T_{01} = T_{10}$ を使うと，

$$T'_{z\bar{z}} = \frac{1}{4}\left(T_{00} + T_{11}\right) \tag{2.30}$$

(c) 同様な手順で

$$T'_{\bar{z}z} = \frac{1}{4}\left(T_{00} + T_{11}\right) \tag{2.31}$$

(d) 同様な手順で

$$T'_{\bar{z}\bar{z}} = \frac{1}{4}\left(T_{00} + 2iT_{10} - T_{11}\right) \tag{2.32}$$

4. (2.7) 式の 1 行上

$$\int j_0(x)dx \to \int j_r(\theta)d\theta \tag{2.33}$$

は多分

$$\int j_0(x)dx \to \frac{1}{2\pi}\int j_r(\theta)d\theta \tag{2.34}$$

のミスプリ

2.3 Free boson, the example

1. ミスプリ (2.17)

   誤: $-\frac{1}{2} : \partial x(z)\partial x(w) :$

   正: $-\frac{1}{2} : \partial x(w)\partial x(w) :$

2.4 Conformal Ward identities

第3章　The central charge and the Virasoro algebra

3.1 The central charge

ミスプリ：(3.4) 式
誤: $-\frac{1}{2} : \partial x(z)\partial x(w) : +i\sqrt{2}\alpha_0\partial^2 x(z)$
正: $-\frac{1}{2} : \partial x(w)\partial x(w) : +i\sqrt{2}\alpha_0\partial^2 x(w)$

3.2 The free fermion

3.3 Mode expansions and the Virasoro algebra

3.4 In- and out-states

3.5 Highest weight states

3.6 Descendant fields

第4章 Kac determinant and unitarity

第5章 Idetification of $m = 3$ with the critical Ising model

第6章　Free bosons and fermions

6.1 Mode expansions

6.2 Twist fields

6.3 Fermionic zero modes

第7章　Free fermions on a torus

第8章　Free bosons on a torus